

Kliem and Török Reply: The Comment by Chen [1] on our investigation of the torus instability (TI) of arched magnetic flux ropes (hereafter KT [2]) states that he has previously solved a more general equation than our Eq. (1)—Eq. (7) in Ref. [3]—(including the effects of non-vanishing plasma beta), using less restrictive assumptions (not assuming constant aspect ratio R/b), so that “the ‘torus instability’ is a limiting case of Ref.” [3]. In addition, it is claimed that (i) the absence in KT of a term that introduces a spatial scale (the footpoint distance S_f of an expanding rope whose ends are anchored in a rigid surface) makes our model conflict with the presence of such a scale in the observations of coronal mass ejections (CMEs) and (ii) the assumption in KT’s Eqs. (8–10) that the ring current I tends to be conserved as a consequence of the rope’s footpoint anchoring is mathematically inconsistent with the ideal-MHD assumption that the magnetic flux Ψ enclosed by the rope is conserved.

The TI is a property of the tokamak equilibrium [4] in the low-beta limit, which is the relevant case in the coronal source regions of CMEs. For $\beta < 1$, the current ring can attain equilibrium only in the presence of an external poloidal field B_{ex} [4]. The instability occurs if B_{ex} decreases sufficiently steeply with increasing major radius R , with a decay index $n = -d \ln B_{\text{ex}} / d \ln R \gtrsim 3/2$ [2, 5]. In Ref. [3] the simplification $B_{\text{ex}} = 0$ was adopted in the starting equations of both, the linear and the nonlinear analysis (Eqs. [7] and [32], respectively), thus *excluding the TI*. (Moreover, since this requires $\beta > 1$ for equilibrium [4], the results in Ref. [3] are largely irrelevant for the explanation of CMEs.)

Unlike Refs. [3, 6, 7], we replaced the integration of the force equation in the b direction by the assumption of self-similar expansion, $R/b = \text{const}$. This is required in order to permit an analytical description (our Eqs. [4–10], which have no analogue in Refs. [3, 6, 7, 8]), and is also a very reasonable simplification, suggested by the observations and justified by the facts that the hoop force depends only logarithmically on R/b and that our resulting description yields qualitative and quantitative agreement with essential CME properties [2, 9, 10]. Chen adopted this assumption recently as well [8], based on the experiences made in fitting his model to CME observations. In integrating the equation for $b(t)$ in Refs. [3, 6, 7], the energy equation was replaced by the polytropic assumption, with the index $\gamma \approx 1.2$ estimated from comparison with observations; this is a level of approximation comparable to ours. Our assumption $R/b = \text{const}$ obviously implies $d^2b/dt^2 \propto d^2R/dt^2$, not $d^2b/dt^2 \simeq 0$ as stated in the Comment.

It is well known that the “standard” MHD equations (e.g., [5]) do not contain an intrinsic length scale. Dimensional lengths enter by prescribing initial or boundary conditions at the application stage, or by specifying the treatment such that these conditions enter already explicitly at an intermediate stage. When our scale-free Eq. (4) is applied to describe CMEs, the footpoint an-

choring of the flux rope, which was not explicitly included in KT, introduces the condition of nearly semicircular flux rope shape at TI onset (which is supported by observations [11] and was suggested in Ref. [7] as well); i.e., $R_0 \approx S_f/2$. With the peak acceleration occurring in the range $R \sim (1.5\text{--}2)R_0 \approx (0.75\text{--}1)S_f$ in the practically most relevant range of n included in Fig. 1 in KT, our description reproduces the observational result of Fig. 1 in the Comment to a very reasonable approximation, the more so if the spread in the position of observed ejecta relative to the magnetic rope axis is taken into account, and it does so in a manner that is more general than the use of a problem-specific inductance in Refs. [6, 7, 8] (which can easily be incorporated in our theory). Contrary to a statement in the Comment, we did not suggest any connection between $I(t)$ and the scale S_f .

I and Ψ can be simultaneously constant in a certain range of R . This is obtained by using these two conditions joint with our general ansatz $B_{\text{ex}}(R) = \hat{B}R^{-n}$ in Eq. (2) and elementary algebra to solve it for b/R , given in Eq. (10) and plotted in Fig. 3 of KT. Using constant I , our Eq. (8) follows exactly from our Eq. (1). The false conclusion in the Comment follows from the inappropriate simplification $L \propto R$, neglecting the $\ln(R/b)$ dependence, which removes b from the equation.

Our estimate of the instability threshold n_{cr} for constant I (Eq. [9]) includes an inconsistency because $R/b = \text{const}$ was used in addition to constant I and Ψ . However, (i) we have expressed that Eqs. (8–9) represent a limiting case, included to demonstrate the *direction* of the effect of constant I , (ii) this estimate does not play any further role in KT, and (iii) the inconsistency of this approximation is made apparent immediately following Eq. (9) and Fig. 2 in KT.

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